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Changing units

Prefixes and conversion factors

Prefix	Abbrev	Conversion factor	...or...	Example		
pico	p	/1,000,000,000,000	$\times 10^{-12}$	2.4 ps	= $2.4 \times 10^{-12}$ s	= 0.000 000 000 002 4 s
nano	n	/1,000,000,000	$\times 10^{-9}$	36 ns	= $36 \times 10^{-9}$ s	= 0.000 000 036 s
micro	$\mu$	/1,000,000	$\times 10^{-6}$	520 $\mu$ s	= $520 \times 10^{-6}$ s	= 0.000 520 s
milli	m	/1,000	$\times 10^{-3}$	58 ms	= $58 \times 10^{-3}$ s	= 0.058 s
kilo	k	$\times 1,000$	$\times 10^3$	7.2 km	= $7.2 \times 10^3$ m	= 7,200 m
mega	M	$\times 1,000,000$	$\times 10^6$	25.6 MHz	= $25.6 \times 10^6$ Hz	= 25,600,000 Hz
giga	G	$\times 1,000,000,000$	$\times 10^9$	111.5 Gb	= $111.5 \times 10^9$ b	= 111,500,000,000 b
tera	T	$\times 1,000,000,000,000$	$\times 10^{12}$	3 Tb	= $3 \times 10^{12}$ b	= 3,000,000,000,000 b

NB. Converting from a memory of kB to kb is complicated because it is sometimes done using SI units and sometimes done using powers of 2. Most hard disk manufacturers these days use SI units (it makes their drives look bigger!). However, due to the confusion David usually explicitly gives  $100 \times 10^3$  B instead of 100 KB. Bit rates (or byte rates) are always use SI units.

## Abbreviations for rates

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There are several different ways to abbreviate 900 bits per second:

- 900 bps
- 900 b/s
- 900  $\text{bs}^{-1}$

The last expression comes from the fact that  $1/\text{s} = \text{s}^{-1}$ .

All of them can be written as  $\frac{900 \text{ b}}{1 \text{ s}}$

and it is this form which is useful for converting between units.

Similarly, 100 metres per second can be abbreviated as:

- 100 m/s
- 100  $\text{ms}^{-1}$

NB. You will not see the abbreviation mps.

## Converting between units

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When working with formulae and solving problems you must make sure that the units are always consistent with each other. In fact, some formulae only work when the quantities are in a particular set of units, eg. Shannon-Hartley Theorem uses Hz and bps. In general, it is easiest to convert all measurements to bits, seconds, metres and Hz.

$$\text{eg. } 5.4 \text{ Mb} = 5.4 \times 10^6 \text{ b} = 5,400,000 \text{ b}$$

$$\text{eg. } 2 \text{ ms} = 2 \times 10^{-3} \text{ s} = 0.002 \text{ s}$$

If you do need to convert between, say, kilobits and Megabits, do it in two steps.

eg. Convert 1.2Mb to kb.

Steps:

(i) Convert to bits as before

$$1.2\text{Mb} = 1.2 \times 10^6 \text{ b} = 1,200,000\text{b}$$

(ii) Convert to kb by dividing by the conversion factor in the above table:

$$1,200,000\text{b} = 1,200,000\text{b} = \frac{1,200,000\text{b}}{10^3 \text{ b/kb}} = 1,200\text{kb}$$

## Converting between rates

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To change one pair of units to another, such as with a rate, convert them one at a time.

eg. Convert 100 km/hr to m/s

Steps:

(i) First convert km to m

$$100 \text{ km/hr} = \frac{100 \text{ km}}{1 \text{ hr}} = \frac{100 \times 10^3 \text{ m}}{1 \text{ hr}} = 100,000 \text{ m/hr}$$

(ii) Then convert hrs to mins

$$100,000 \text{ m/hr} = \frac{100,000 \text{ m}}{1 \text{ hr}} = \frac{100,000 \text{ m}}{60 \text{ mins}} = \frac{1,666.67 \text{ m}}{1 \text{ min}} = 1,666.67 \text{ m/min}$$

(iii) Then convert mins to secs

$$1,666.67 \text{ m/min} = \frac{1,666.67 \text{ m}}{1 \text{ min}} = \frac{1,666.67 \text{ m}}{60 \text{ secs}} = \frac{27.78 \text{ m}}{1 \text{ sec}} = 27.78 \text{ m/s}$$

## Propagation delay time revisited

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As mentioned in Week 4, propagation delay time can be calculated as follows:

$$t_{\text{prop}} = \frac{\text{cable length}}{\text{propagation speed}}$$

but you must make sure the cable length and propagation speed are in the same units.

eg. What is the propagation delay time for a 10km coax cable ( $v = 2.4 \times 10^8 \text{ ms}^{-1}$ )?

Steps:

(i) Convert the length to m to match up with m/s:

$$10\text{km} = 10 \times 10^3 \text{m} = 10,000\text{m}$$

(ii) Calculate  $t_{\text{prop}}$  using the formula:

$$t_{\text{prop}} = \frac{10,000 \text{ m}}{2.4 \times 10^8 \text{ m/s}} = 4.17 \times 10^{-5} \text{ s}$$

## Data frame transmission time revisited

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As mentioned in Week 2, data frame transmission time can be calculated as follows:

$$t_{\text{data}} = \frac{\text{frame size}}{\text{frame transmission rate}}$$

but you must make sure the frame size and frame transmission rate are in the same units.

eg. What is the data frame transmission time for a 3.6Mb frame over a 1.2Mbps link?

Steps:

(i) There is no need to convert both lots of Mb to b, but you could.

(ii) Calculate  $t_{\text{data}}$  using the formula:

$$t_{\text{data}} = \frac{3.6 \text{ Mb}}{1.2 \text{ Mb/s}} = 3\text{s}$$

## Decibels

### Definition

The decibel (dB) is a logarithmic unit used to describe a ratio<sup>1</sup>:

$$\text{ratio}_{\text{dB}} = 10 \log_{10} \text{ratio}$$

Common ratios measured in decibels are two power levels, two sound pressures and two voltages.

### Converting from SNR to $\text{SNR}_{\text{dB}}$

In this subject you will use decibels to measure the signal to noise ratio (SNR):

$$\text{SNR} = \frac{\text{Average signal power}}{\text{Average noise power}}$$

Expressing the signal to noise ratio in decibels ( $\text{SNR}_{\text{dB}}$ ):

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR}$$

NB. SNR does not mean  $S \times N \times R$ , it is simply an abbreviation.

eg. If the average signal power is twice the average noise power, what is the signal to noise ratio? What is the signal to noise ratio in decibels?

$$\text{SNR} = 2$$

$$\begin{aligned} \text{SNR}_{\text{dB}} &= 10 \log_{10} \text{SNR} \\ &= 10 \log_{10} 2 \\ &= 10 \times 0.301 \\ &= 3.01 \text{ dB} \\ &\approx 3 \text{ dB} \end{aligned}$$

eg. If the average noise power is twice the average signal power, what is the signal to noise ratio? What is the signal to noise ratio expressed in dB?

$$\text{SNR} = \frac{1}{2} = 0.5$$

$$\begin{aligned} \text{SNR}_{\text{dB}} &= 10 \log_{10} \text{SNR} \\ &= 10 \log_{10} 0.5 \\ &= 10 \times -0.301 \\ &= -3.01 \text{ dB} \\ &\approx -3 \text{ dB} \end{aligned}$$

<sup>1</sup> For more on ratios go back to the Maths Worksheet for Week 2 Tutorial

## Converting from SNR<sub>dB</sub> to SNR

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We can rearrange the formula above to convert back from dB.

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR}$$

$$\frac{\text{SNR}_{\text{dB}}}{10} = \log_{10} \text{SNR}$$

$$\boxed{\text{SNR} = 10^{\left(\frac{\text{SNR}_{\text{dB}}}{10}\right)}}$$

eg. What signal to noise ratio corresponds to a signal to noise ratio of 3dB?

$$\text{SNR} = 10^{\left(\frac{\text{SNR}_{\text{dB}}}{10}\right)}$$

$$= 10^{\left(\frac{3}{10}\right)}$$

$$= 10^{0.3}$$

$$= 1.995$$

$$\approx 2$$

## Some useful dB values

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You can verify the following signal to noise ratios yourselves:

SNR	0.000001	0.01	0.1	0.5	1	2	10	100	1,000,000
SNR <sub>dB</sub>	-60	-20	-10	-3	0	3	10	20	60

Note:

- A negative SNR<sub>dB</sub> means the average signal is smaller than the average noise
- A SNR<sub>dB</sub> of zero means the average signal is equal to the average noise
- A positive SNR<sub>dB</sub> means the average signal is larger than the average noise
- Very large and very small ratios are common for ratios such as SNR. The SNR<sub>dB</sub> will be comparatively small, which is why SNR is often expressed in decibels.

See Appendix 3A in Stallings for more on Decibels

# Shannon-Hartley Theorem

## Formula for Capacity

Given the bandwidth and signal to noise ratio of a channel, the maximum theoretical capacity of the channel under ideal conditions is given by the formula:

$$C = B \log_2(1 + \text{SNR})$$

where:

$B$  = Bandwidth of the channel measured in Hertz<sup>2</sup> (Hz)

$C$  = Channel/Information Capacity measured in bits per second (bps)

SNR = Noise Ratio with no units (ie. SNR not SNR<sub>dB</sub>)

The first step to using the Shannon-Hartley Theorem is to convert to required units.

eg. What is the maximum theoretical capacity of a channel with a bandwidth of 5.6 kHz and a signal to noise ratio of 22 dB?

Steps:

(i) Convert bandwidth to Hz

$$5.6 \text{ kHz} = 5.6 \times 10^3 \text{ Hz}$$

$$= 5600 \text{ Hz}$$

(ii) Convert SNR<sub>dB</sub> to SNR

$$\text{SNR} = 10^{(\text{SNR}_{\text{dB}}/10)}$$

$$= 10^{(22/10)}$$

$$= 10^{2.2}$$

$$= 158.5$$

(iii) Using the formula

$$C = B \log_2(1 + \text{SNR})$$

$$= 5600 \times \log_2(1 + 158.5)$$

$$= 5600 \times 3.322 \times \log_{10} 159.5$$

$$= 40,978 \text{ bps or } 40.98 \text{ kbps}$$

<sup>2</sup> Hz is equivalent to cycles/s

## Solving for Bandwidth

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The formula for the Shannon-Hartley Theorem can be rearranged to find the minimum theoretical bandwidth required to produce a specified channel capacity given the signal to noise ratio.

eg. Ideally, what is the minimum bandwidth required for a common channel that has a bit rate of 380 kbps, and a SNR of 15 dB?

Steps:

(i) Convert capacity to bps

$$\begin{aligned}380\text{ kbps} &= 380 \times 10^3 \text{ bps} \\ &= 380000 \text{ bps}\end{aligned}$$

(ii) Convert  $\text{SNR}_{\text{dB}}$  to SNR

$$\begin{aligned}\text{SNR} &= 10^{(\text{SNR}_{\text{dB}}/10)} \\ &= 10^{(15/10)} \\ &= 10^{1.5} \\ &= 31.623\end{aligned}$$

(iii) Rearrange formula and express answer in correct units

$$\begin{aligned}C &= B \log_2(1 + \text{SNR}) \\ 380,000 &= B \times \log_2(1 + 31.623) \\ 380,000 &= B \times 3.322 \times \log_{10} 32.623 \\ &= B \times 5.028 \\ \frac{380,000}{5.028} &= B \\ B &= 75,577.87 \text{ Hz or } 75.58 \text{ kHz}\end{aligned}$$

## Solving for SNR

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The formula for the Shannon-Hartley Theorem can also be rearranged to find the minimum theoretical SNR required to produce a specified channel capacity given the bandwidth.

eg. Ideally, what is the minimum SNR required for a channel that has a bit rate of 344 kbps, and a bandwidth of 533kHz?

Steps:

(i) Convert capacity to bps

$$\begin{aligned}344\text{kbps} &= 344 \times 10^3 \text{ bps} \\ &= 344,000 \text{ bps}\end{aligned}$$

(ii) Convert bandwidth to Hz

$$\begin{aligned}533\text{kHz} &= 533 \times 10^3 \text{ kHz} \\ &= 533,000 \text{ Hz}\end{aligned}$$

(iii) Rearrange the formula and express answer in correct units

$$C = B \log_2(1 + \text{SNR})$$

$$344,000 = 533,000 \times 3.322 \times \log_{10}(1 + \text{SNR})$$

$$344,000 = 1,770,626 \times \log_{10}(1 + \text{SNR})$$

$$\frac{344,000}{1,770,626} = \log_{10}(1 + \text{SNR})$$

$$0.1943 = \log_{10}(1 + \text{SNR})$$

$$10^{0.1943} = 1 + \text{SNR}$$

$$1.564 = 1 + \text{SNR}$$

$$\text{SNR} = 0.564$$

(iv) If dB was preferred:

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR}$$

$$= 10 \log_{10} 0.564$$

$$= 10 \times -0.25$$

$$= -2.5 \text{ dB}$$

## Questions

- Convert the following quantities to the required units:
  - 567ms to s
  - 1.4kHz to Hz
  - 2Mbps to kbps
  - 2987Hz to kHz
- Find the propagation delay time for:
  - a 300m optical fibre.
  - a 10km coax cable.
- Find the data frame transmission time for:
  - a 500 bit frame over a 200Mbps link.
  - a 1.2 kb frame over a 600 Mbps link.
- If  $\text{SNR}_{\text{dB}}=4\text{dB}$ , what is the SNR.
- Express a SNR of 240 in decibels.
- Find the maximum capacity of a channel with:
  - a 300 Hz bandwidth and a signal to noise ratio of 70.
  - a bandwidth of 5MHz and a signal to noise ratio of 20db.
- Find the ideal bandwidth which would allow allow a channel to operate at:
  - 9.6kbps if the signal to noise ratio is 1200.
  - 1.1Mbps if the signal to noise ratio is 35db.
- Find the minimum signal to noise ratio required for a channel that has:
  - a bit rate of 900 kbps and a bandwidth of 1.2MHz.
  - a bit rate of 700 kbps, and a bandwidth of 800kHz. Convert your answer to dB.

## Answers

- 0.567 ms
  - 1,400 Hz
  - 2,000 kbps
  - 2.987 kHz
- $0.00000147 \text{ s} = 1.47\mu\text{s}$
  - $0.000042 \text{ s} = 42\mu\text{s}$
- $0.0000025 \text{ s} = 2.5 \mu\text{s}$
  - $0.000002 \text{ s} = 2\mu\text{s}$
- 2.512
- 23.80 dB
- $1845\text{bps} = 1.845\text{kbps}$
  - $33291057.41\text{bps} = 33.3\text{Mbps}$
- 938.39Hz
  - 94.603 kHz
- 0.682
  - $0.834 = -0.788\text{dB}$