Maths Worksheet for Tutorial 2 | ITB627 Week 2

Maths Worksheet

for Tutorial 2

ITB627

Week 2

Topics

- **Symbols**
- **Ratios – *a***
- **Utilisation equations**
- **Logs – an introduction**
- **Reading graphs – utilisation**
  - Axes
  - Log scale
  - Different uses for graphs
  - Graphs of more than two variables

Symbols

Symbol	Meaning	Example
$\approx$	approximately equal to	$6.00001 \approx 6$
$<$	less than	$1 < 2$
$>$	greater than	$2 > 1$
$\leq$	less than or equal to	$1 \leq 1$
$\geq$	greater than or equal to	$2 \geq 1$
$\lceil \; \rceil$	round up	$\lceil 12.243 \rceil = 13$
$\lfloor \; \rfloor$	round down	$\lfloor 12.243 \rfloor = 12$
$\sum$	sum together	$\sum_{i=1}^{3} i = 1 + 2 + 3$
$\prod$	multiply together	$\prod_{i=1}^{3} i = 1 \times 2 \times 3$

Ratios - a

The 'ratio' of the propagation delay time, t_{prop}, to the data frame transmission time, t_{data}, is represented mathematically as:

$$\boxed{a = \frac{t_{prop}}{t_{data}}}.$$

NB. t_{prop} is called the *numerator* of the ratio and t_{data} is called the *denominator* of the ratio.

There are several things to point out about this special ratio a.

- **The ratio a has no units, because both t_{prop} ratio and t_{data} are times.**
  The units in the numerator (t_{prop}) cancel with the units in the denominator (t_{data}).
  eg. When $t_{prop} = 1\text{s}$ and $t_{data} = 100\text{ms}$,
  $$a = \frac{t_{prop}}{t_{data}} = \frac{1\text{ s}}{100\text{ ms}} = \frac{1\text{ s}}{100 \times 10^{-3}\text{ s}} = 10$$
  NB. A future worksheet will cover converting units.

- **The propagation delay time is a times the data frame transmission time.**
  This can be seen by rearranging $a = \frac{t_{prop}}{t_{data}}$ to get $t_{prop} = a \times t_{data}$.
  eg. When $t_{prop} = 10\text{ms}$ and $t_{data} = 1\text{ms}$,
  $$a = \frac{t_{prop}}{t_{data}} = \frac{10\text{ ms}}{1\text{ ms}} = 10$$

- **Many different values of t_{prop} and t_{data} give the same ratio a.**
  This follows from the last point.
  eg. Other combination of t_{prop} and t_{data} resulting in a ratio of $a = 10$ are:
  1. $t_{prop} = 50\text{ms}$ and $t_{data} = 5\text{ms}$
  2. $t_{prop} = 940\mu\text{s}$ and $t_{data} = 94\mu\text{s}$

- **The sign of a has meaning.**
  Because both the numerator (t_{prop}) and denominator (t_{data}) of the ratio a are positive:

$0 < a < 1$	implies	$t_{prop} < t_{data}$	ie. t_{prop} is faster than t_{data}
$a = 1$	implies	$t_{prop} = t_{data}$	ie. t_{prop} is the same as t_{data}
$a > 1$	implies	$t_{prop} > t_{data}$	ie. t_{prop} is slower than t_{data}

eg. If $t_{prop} = 2\text{ ms}$ and $t_{data} = 8\text{ ms}$, then $a = 0.25$
eg. If $t_{prop} = 50\text{ ms}$ and $t_{data} = 50\text{ ms}$, then $a = 1$
eg. If $t_{prop} = 100\text{ ms}$ and $t_{data} = 5\text{ ms}$, then $a = 20$

Utilisation equations

Utilisation equations are used to calculate the utilisation of a link, U, given attributes of the link such as the window size, W, the ratio of the propagation delay time to the data frame transmission time, a, and the probability of sending a data frame in error, p_e.

For a sliding window flow control, utilisation is calculated using the following *piecewise* equation:

$$U = \begin{cases} 1 & W \ge 1+2a \\ \dfrac{W}{1+2a} & W < 1+2a \end{cases}$$

What this means is utilisation is calculated from one of two equations depending on the relationship between W and a. Simplifying the utilisation equation to:

$$U = \begin{cases} \text{rule 1} & \text{condition 1} \\ \text{rule 2} & \text{condition 2} \end{cases}$$

the steps for calculating utilisation are:

Step 1. Determine which condition is satisfied.

Step 2. Apply the corresponding rule ie.

if	condition 1	then	use rule 1
else if	condition 2	then	use rule 2

eg. Calculate the utilisation for a sliding window flow control with W=2, t_{prop}=10ms & t_{data}=1ms.

Step 1.
(i) $a = \dfrac{t_{\text{prop}}}{t_{\text{data}}} = \dfrac{10\text{ms}}{1\text{ms}} = 10$
(ii) $1+2a = 1+2\times 10 = 21$
(iii) Is $W \ge 1+2a$? No
(iv) Is $W < 1+2a$? Yes

Step 2. $U = \dfrac{W}{1+2a} = \dfrac{2}{21} = 0.0952$ or 9.52%

Logs – an introduction

Logarithms (logs) are used to write powers in a different form. So the other way of writing

$$\boxed{10^2 = 100}$$

where 10 is the *base* and 2 is the *exponent*, is

$$\boxed{\log_{10} 100 = 2}$$

which is read "log of 100 to base 10 is 2".

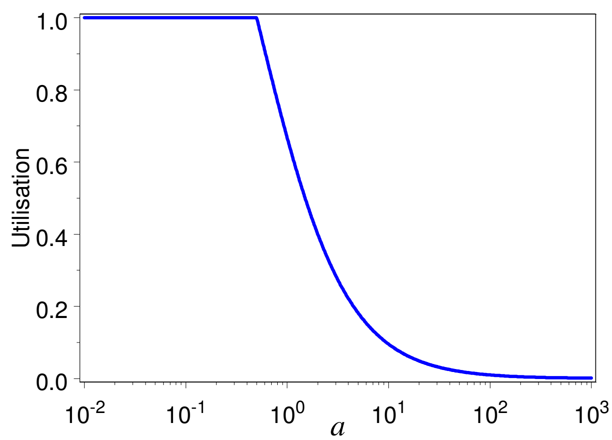
You can think of logs as: what power do I need to raise 10 to, to get 100? The power of 2.

The other base you will work with is base 2, eg. $\log_2 8 = 3$, or $2^3 = 8$. NB. While any positive base is legitimate, you will only be working with base 10 and base 2.

eg. $\log_{10} 0.01 = -2$, $\log_{10} 0.1 = -1$, $\log_{10} 1 = 0$, $\log_{10} 10 = 1$, $\log_{10} 100 = 2$.
You can verify these values using you calculator.

Reading graphs – utilisation

Equations defining the relationship between two or more variables are often plotted on graphs. The following graph plots the relationship between utilisation and a for a sliding window flow control with a window size of 2.



Sliding window performance (W=2)

NB. There are three *variables* in the utilisation equations for a sliding window flow control: U, a, and W. To see how two of the variables, say U and a, change together, we need to fix the other variable, W, as above.

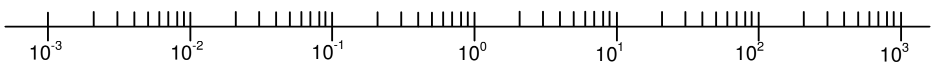
Axes

The first step to reading a graph involves looking at the axes:

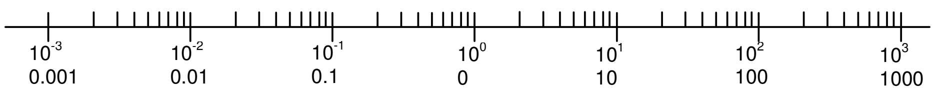
1. **Look which unknowns are plotted on each axis:** In this case U and a. ie. not W.
2. **Look at the units of each axis:** Neither U or a have units. NB. U is not given as a percentage.
3. **Look at the scale of each axis:** U is straightforward to read. However, a is plotted on a *log scale* which requires special care.

Log scale

When you see an axis on a graph such as:



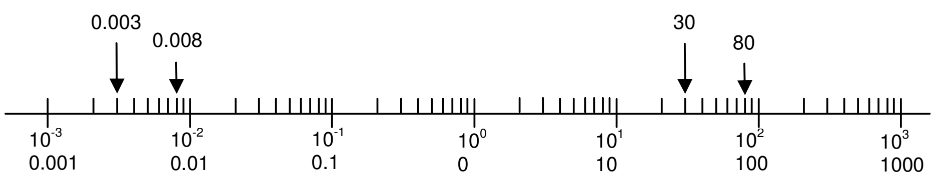
what is actually being plotted is:



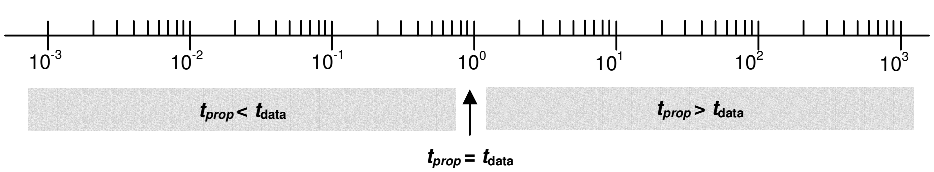
On the graph there is the same distance between 0.001 and 0.01 as there is between 100 and 1000; whereas *in reality* the numbers 0.001 and 0.01 are much closer together than the numbers 100 and 1000.

Finding a number between two of the major notches (0.001, 0.01, 0.1, 1, 10, 100 & 1000) involves counting off the smaller, unevenly spaced notches between them.

eg. Where are the values 0.003, 0.008, 30 and 80 located?



If the numbers being plotted were values of the ratio a, then it is useful to note the following regions on the graph:



Different uses for graphs

Each point on the line (and there are an infinite number of points) plotted in the graph on page 4 represents a pair of values for a and U which satisfy the utilisation equation for a sliding window flow control with a window size of 2. So the graph has several uses:

1. Finding the value of one variable which corresponds to the value of the other variable.

eg. Use the graph on page 4 to do the example on page 3. ie. Calculate the utilisation for a sliding window flow control with W=2, t_{prop}=10ms & t_{data}=1ms.

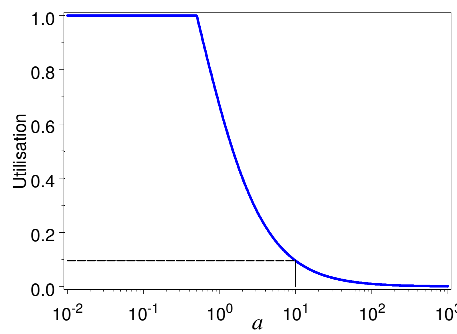
Steps:

(i) Calculate $a = \frac{t_{\text{prop}}}{t_{\text{data}}} = \frac{10\text{ms}}{1\text{ms}} = 10$

(ii) Find $a = 10$ on the a-axis.

(iii) Draw a line from the a-axis up to the curve and then across the U-axis.

(iv) Read off the value of $U \approx 0.1$ or 10% on the U-axis. It is hard to calculate U exactly from the graph.



NB. There was no need to find $1+2a$ etc. because the only thing you need to find U (given the graph of W=2) is a.

2. Finding a range of values for one variable which satisfy a range of values for the other variable.

eg. For a sliding window flow control, what propagation delay times and data frame transmission times would give a utilisation of at least 90%?

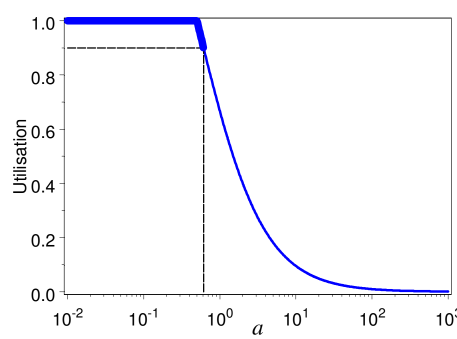
Steps:

(i) Find $U = 0.9$ on the U-axis.

(ii) Draw a line from the U-axis across to the curve, and then down to the a-axis.

(iii) Read off the value of $a \approx 0.6$ on the a-axis.

(iv) All values of a *less than* (not greater than) $a \approx 0.6$ give values of U *greater than* 0.9.

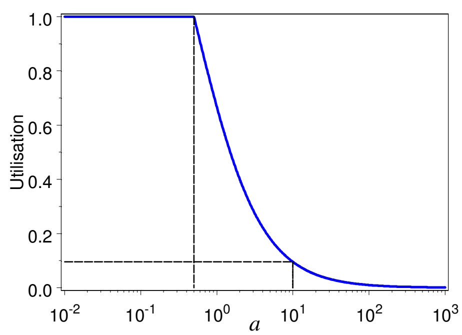


(v) In words: Utilisation will be greater than 90% when the propagation delay time is less than 0.6 times the data frame transmission time.

3. Making more general statements about the relationship between the two variables.

eg. Describe how the ratio of the propagation delay time to the data frame transmission time affects utilisation for a sliding window flow control with window size 2.

Steps:



(i) The most obvious feature of the curve is a sudden change in the relationship between a and U when $a = 0.5$.

(ii) When $a < 0.5$, $U = 1$.

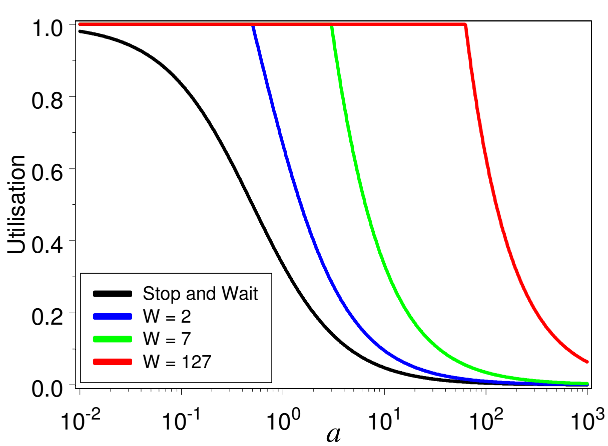
(iii) When $a > 0.5$, U drops off sharply. (NB. This is the optimal window size $W = 1 + 2a$).

(iv) Once $a > 10$ the utilisation has fallen to 0.1 and then decreases more gradually to 0.0.

(v) In words: When the propagation delay time is less than half the data frame transmission time, utilisation is 100%. There is a steep drop-off in utilisation from 100% to 10% as the ratio of the propagation delay time to the data frame transmission time increases from a half to ten. The utilisation then drops off slowly and approaches 0% when the propagation delay time is much larger than the data frame transmission time.

Graphs of more than two variables

The last section looked at the relationship between *two* variables. To study the relationship between *three* variables, you can either add a third axis[1] or draw multiple curves on a two-dimensional graph. The graph below plots the relationship between a, U and (selected values of) W for the sliding window flow control.



Sliding window performance

[1] Three-dimensional graphs will be covered later in a future worksheet.

Questions

1. Find the utilisation for a link using sliding window flow control with parameters:
   (i) W=7, t_{prop}=22 ms, t_{data}=2ms
   (i) W=2, t_{prop}=1ms, t_{data}=10ms
2. Find the average utilisation for a link with Go-Back-N ARQ and:
   (ii) W=127, t_{prop}=60ms, t_{data}=5ms, p_e=1%
   (ii) W=127, t_{prop}=400ms, t_{data}=3ms, p_e=10%
3. Find the average utilisation for a link with selective reject ARQ and:
   (iii) W=4, t_{prop}=26ms, t_{data}=9ms, p_e=1%
   (iii) W=8, t_{prop}=6ms, t_{data}=4ms, p_e=0.1%
4. Verify your answers to questions 1 and 2 using the graphs in your lecture notes.
5. Find the optimal window sizes for the links described in question 1.

Answers

1.	(a) U≈30.4%	(b) U≈100%
2.	(a) U≈79.8%	(b) U≈3.1%
3.	(a) U≈58.4%	(b) U≈99.9%
5.	(a) 23	(b) 2